

# Homework 4

## Solution Key (Revenue-Share HHI, CR2, CR4)

[Download HW 4 solution file here](#)

### Definitions

Let firm  $i$  have revenue  $R_i = p_i \cdot q_i$ .

Let total market revenue be  $VM$  (column  $VM$ ).

Define **revenue share** as:

$$sh_i = \frac{R_i}{VM} = \frac{p_i q_i}{VM}$$

- **Herfindahl–Hirschman Index (HHI, 0–10,000 scale)**

$$HHI = \sum_{i=1}^N (100 \cdot sh_i)^2$$

- **Concentration Ratio ( $CR_k$ )**

$$CR_k = \sum_{j=1}^k sh_{(j)} \quad \text{where } sh_{(1)} \geq sh_{(2)} \geq \dots$$

- **CR2:** sum of top two revenue shares
- **CR4:** sum of top four revenue shares

Use revenue shares for **all** calculations:

$$sh_i = \frac{p_i q_i}{VM}$$

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### Mapping to Excel Columns

Per month (row):

#### Revenues

- $r1 = p1 \cdot q1$  (Jeminy)

- $r_2 = p_2 * q_2$  (Cousins)
- $r_3 = p_3 * q_3$
- $r_4 = p_4 * q_4$
- $r_5 = p_5 * q_5$
- **All Others:**  
 $r_{\text{others}} = \text{MAX}(0, \text{VM} - (r_1 + r_2 + r_3 + r_4 + r_5))$

### Shares

- $sh_i = r_i / \text{VM}$  for each of the 5 named firms (+ Others if  $r_{\text{others}} > 0$ )

**Excel tips** (replace `shares_range` with the cell range of shares):

- $\text{HHI} = \text{SUMPRODUCT}((100 * \text{shares\_range})^2)$
- $\text{CR}_2 = \text{SUM}(\text{LARGE}(\text{shares\_range}, \{1, 2\}))$
- $\text{CR}_4 = \text{SUM}(\text{LARGE}(\text{shares\_range}, \{1, 2, 3, 4\}))$

Do **not** use QM for HHI/CR — we are using **revenue** shares, not quantity shares.

### Graphs Over Time

Create a Year–Month column and compute monthly **HHI**, **CR2**, and **CR4** using revenue shares. Produce three line charts: HHI, CR2, and CR4 vs. `date`.

### Interpretation Thresholds

DOJ/FTC HHI screens (0–10,000 scale):

- **Unconcentrated:**  $\text{HHI} < 1500$
- **Moderately concentrated:** 1500–2500
- **Highly concentrated:**  $> 2500$

**CR** rules of thumb:

- **CR4 < 40%** → fragmented / competitive
- **CR4 40–60%** → loose oligopoly
- **CR4 > 60%** → tight oligopoly
- **High CR2 (> 40–50%)** → strong duopoly tendency

Use both **levels** and **trends** to describe market structure.

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## Answering Management's Questions

- **HHI over time:** Report the level and trend. Compare to thresholds above.
- **CR2/CR4 over time:** Highlight concentration patterns. High/stable CRs indicate persistent market power.
- **Perfect competition?**
  - Only plausible if HHI is low (<1500) and CR4 < 40%.
  - Otherwise, the market is likely **oligopolistic**, not perfectly competitive.
- **Cousins' position:**  
Report

$$sh_{\text{Cousins}} = \frac{p_2 q_2}{VM}$$

each month, note if it is a top-2 or top-4 firm, and whether its share is increasing or decreasing over time.

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## Common Pitfalls

- Forgetting **Others** when  $VM > \sum(p_i q_i)$  → underestimates concentration.
- Mixing units (quantities instead of revenues).
- Shares must sum to 1 for accurate results.
- HHI/CR should always be calculated **monthly**, then graphed over time.