Equilibrium Displacement Models

Elasticities and Forecasting

Elasticities are useful tools in applied economics. They tell us:

- whether a tax on a specific good will raise large or small revenues,
- which goods are the greatest competitors, and
- how prices and quantities may change in response to shocks.

For example:

- How does the supply of pork decreases due to tougher regulations on manure treatment and disposal?
- How much will the demand for pork increase due to an increase in the price of beef?

We do not need to know the *exact* supply and demand curves. Estimates of elasticities are usually sufficient.

By incorporating elasticities into an **equilibrium displacement model (EDM)**, we can calculate price and quantity changes resulting from outside shocks.

Endogenous and Exogenous Variables

Equilibrium displacement models are used to assess how *endogenous variables* respond to changes in *exogenous variables*.

In an EDM, all variables are grouped as either **endogenous** or **exogenous**.

- **Endogenous variables**: Determined *within* the model (price and quantity).
- **Exogenous variables**: Determined *outside* the model and taken as given.
    - Examples: input prices, technology, consumer income, population, tastes, and prices of related goods.

Shocks to exogenous variables shift supply and/or demand curves. The market then finds a new equilibrium where supply equals demand.

Formalizing the Model

Demand Curve

Step 1. Start with the elasticity definition

For demand, the own-price elasticity is defined as:

$$E_D = \frac{\%\Delta Q_D}{\%\Delta P}$$

This formula says: "a 1% change in price leads to E_D percent change in quantity demanded." Remember that E_D is typically **negative** because price and quantity demanded move in opposite directions.

Step 2. Rearrange to express quantity changes

Multiply both sides by $\%\Delta P$ to solve for the percent change in quantity demanded:

$$\%\Delta Q_D = E_D \cdot (\%\Delta P)$$

This equation tells us how much quantity demanded changes in response to a given price change, holding everything else constant.

Step 3. Add in demand shocks

Markets can also shift for reasons other than price (income, tastes, population, etc.).
We capture these **exogenous demand shifts** with a new term, S_D:

$$\%\Delta Q_D = E_D(\%\Delta P) + S_D$$

- $\%\Delta Q_D$: percent change in quantity demanded
- $\%\Delta P$: percent change in price
- E_D: own-price elasticity of demand (negative)
- S_D: demand shock (e.g., if incomes rise and demand increases by 7%, then $S_D = 7$)

Takeaway: This step-by-step process shows how we move from the basic elasticity definition to an equation that includes both **price effects** and **shifts in demand.**

Supply Curve

Step 1. Start with the elasticity definition

For supply, the own-price elasticity is defined as:

$$E_S = \frac{\%\Delta Q_S}{\%\Delta P}$$

This formula says: "a 1% change in price leads to E_S percent change in quantity supplied."
Unlike demand, E_S is typically **positive** because price and quantity supplied move in the same direction.

Step 2. Rearrange to express quantity changes

Multiply both sides by $\%\Delta P$ to solve for the percent change in quantity supplied:

$$\%\Delta Q_S = E_S \cdot (\%\Delta P)$$

This equation captures how much quantity supplied changes in response to a price change, holding everything else constant.

Step 3. Add in supply shocks

Supply can also shift due to exogenous factors (e.g., technology, input costs, regulations).
We capture these **exogenous supply shifts** with a new term, S_S:

$$\%\Delta Q_S = E_S(\%\Delta P) + S_S$$

- $\%\Delta Q_S$: percent change in quantity supplied
- $\%\Delta P$: percent change in price
- E_S: own-price elasticity of supply (positive)
- S_S: supply shock (e.g., if new technology lowers costs and supply increases 5%, then $S_S = 5$)

Solving for the Equilibrium Price

Goal: Solved for the endogenous price change.

1. In equilibrium, quantity supplied equals quantity demanded:

$$\%\Delta Q_S = \%\Delta Q_D$$

2. Substitute in the supply and demand equations:

$$E_S(\%\Delta P) + S_S = E_D(\%\Delta P) + S_D$$

3. Rearrange terms:

$$(E_S - E_D)(\%\Delta P) = S_D - S_S$$

4. Solve for the endogenous variable, price:

$$\%\Delta P = \frac{S_D - S_S}{E_S - E_D}$$

Intuition

- The denominator $(E_S - E_D)$ is positive since $E_S > 0$ and $E_D < 0$.
- If $S_D > 0$ (a positive demand shock, e.g., higher incomes) and $S_S = 0$, then $\%\Delta P > 0 \rightarrow$ price rises.
- If $S_S > 0$ (a positive supply shock, e.g., lower production costs) and $S_D = 0$, then $\%\Delta P < 0$ $\rightarrow$ price falls.

Takeaway: With both curves expressed in elasticity form, we can solve for how price responds to shocks. Once $\%\Delta P$ is known, we substitute back into either curve to find the equilibrium change in quantity.

Solving for the Equilibrium Quantity

Once we know the equilibrium price change $\%\Delta P$ is found, we can solve for the change in quantity by substituting back into either curve:

- Supply:

$$\%\Delta Q_S = E_S(\%\Delta P) + S_S$$

- Demand:

$$\%\Delta Q_D = E_D(\%\Delta P) + S_D$$

Both give the same $\%\Delta Q$ in equilibrium.

Solving for the Equilibrium Quantity

Goal: Use that price change to back out the endogenous quantity change.

To find $\%\Delta Q$, we substitute the equilibrium price change into either the supply or demand equation.

1. Using the supply curve:

$$\%\Delta Q_S = E_S(\%\Delta P) + S_S$$

Substitute the equilibrium price solution:

$$\%\Delta Q_S = E_S\left(\frac{S_D - S_S}{E_S - E_D}\right) + S_S$$

2. Using the demand curve:

$$\%\Delta Q_D = E_D(\%\Delta P) + S_D$$

Substitute the equilibrium price solution:

$$\%\Delta Q_D = E_D\left(\frac{S_D - S_S}{E_S - E_D}\right) + S_D$$

3. Confirm consistency:

In equilibrium,

$$\%\Delta Q_S = \%\Delta Q_D = \%\Delta Q.$$

If the two do not match, it signals an algebra error in your calculations.

Takeaway: First solve for $\%\Delta P$ using equilibrium, then plug it into either curve to get $\%\Delta Q$. Both equations must agree at the new equilibrium.

Proof

Once we know the equilibrium price change $\%\Delta P$, we can solve for the change in quantity.

1. Substitute $\%\Delta P$ into the supply curve

Recall the supply curve:

$$\%\Delta Q_S = E_S(\%\Delta P) + S_S$$

Plug in the solved value for $\%\Delta P$:

$$\%\Delta Q_S = E_S\left(\frac{S_D - S_S}{E_S - E_D}\right) + S_S$$

2. Or substitute into the demand curve

Recall the demand curve:

$$\%\Delta Q_D = E_D(\%\Delta P) + S_D$$

Substitute the same $\%\Delta P$:

$$\%\Delta Q_D = E_D\left(\frac{S_D - S_S}{E_S - E_D}\right) + S_D$$

3. Confirm consistency
In equilibrium, both give the same answer:

$$\%\Delta Q_S = \%\Delta Q_D = \%\Delta Q$$

If they don't match, a calculation error has been made.

Intuition

- A positive demand shock ($S_D > 0$) raises price and increases equilibrium quantity.
- A positive supply shock ($S_S > 0$) lowers price but still increases equilibrium quantity.
- The exact size of the change depends on how responsive supply and demand are (their elasticities).

General Steps for an EDM

1. **Identify shocks:** Determine S_S and S_D. At least one must be nonzero.
2. **Specify supply curve:**

$$\%\Delta Q_S = E_S(\%\Delta P) + S_S$$

3. **Specify demand curve:**

$$\%\Delta Q_D = E_D(\%\Delta P) + S_D$$

4. **Solve for price:** Set $\%\Delta Q_S = \%\Delta Q_D$ and solve for $\%\Delta P$.
5. **Solve for quantity:** Substitute $\%\Delta P$ into either equation to calculate $\%\Delta Q$.

Example: Impacts of Manure Regulations in the Pork Market

Pork producers are facing tougher regulations on how they manage and treat hog manure.

These regulations raise production costs, which shifts the supply curve **leftward**, raising price and lowering quantity.

We quantify these changes using elasticities and an equilibrium displacement model (EDM).

> **Note**
>
> This example assumes **long-run elasticities** ($E_S = 2.15$, $E_D = -1.96$).
> To examine the short-run impacts, simply replace these with short-run elasticities.

Step 1. Express the cost increase as a percent of the current price

Suppose the current hog price is \$50/cwt, and the new regulations increase production costs by \$1/cwt.

$$P = \$50 \text{ per cwt}$$
$$\Delta C = \$1 \text{ per cwt}$$

Calculate the cost increase as a percentage of the current price:

$$\frac{\Delta C}{P} = \frac{\$1}{\$50} = 0.02 = 2\%$$

Interpretation: Paying \$1 more per cwt is like receiving \$1 less per cwt in revenue. From the producer's perspective, this is an **effective price decrease** of 2%:

$$\%\Delta P_{\text{effective}} = -2\%.$$

Note that this is the change in price received by the producer as a result of the regulation. This is *not* the change in the equilibrium price.

Step 2. Convert the effective price change into a supply shock

Goal: Calculate how much supply would shift if producers suddenly faced a 2% lower effective price, before price adjusts in the market.

The **supply elasticity** tells us how responsive producers are to a change in price:

$$E_S = \frac{\%\Delta Q_S}{\%\Delta P}.$$

Rearrange to solve for the percent change in quantity supplied:

$$\%\Delta Q_S = E_S \times (\%\Delta P).$$

This equation says: if the effective price producers receive changes by some percent, then quantity supplied will change by elasticity × that percent. In our case, the regulation is equivalent to a **2% fall in effective price**. It is also important to know that the (long-run) supply elasticity $E_S = 2.15$

Substitute the effective price change from Step 1 (-2%) and the long-run supply elasticity $E_S = 2.15$:

$$S_S = E_S \times (\%\Delta P_{\text{effective}})$$
$$S_S = 2.15 \times (-2)$$
$$S_S = -4.3\%$$

::: {.callout-important} Note the change in notation where we replace $\%\Delta Q_S$ with S_S above. Because we are only interested in the portion of the change in quantity caused directly by the regulation before the market adjusts price, we replaced $\%\Delta Q_S$ with S_S. :::

Interpretation: A 2% decrease in effective price leads to a 4.3% decrease in supply.
This is the **exogenous supply shock**: $S_S = -4.3\%$.
Because nothing changed on the demand side, $S_D = 0$.

Step 3. Write the EDM equations

Supply:

$$\%\Delta Q_S = E_S(\%\Delta P) + S_S = 2.15(\%\Delta P) - 4.3$$

Demand:

$$\%\Delta Q_D = E_D(\%\Delta P) + S_D = -1.96(\%\Delta P) + 0$$

Step 4. Impose Equilibrium (Set supply equal to demand)

In equilibrium, $\%\Delta Q_S = \%\Delta Q_D$:

$$2.15(\%\Delta P) - 4.3 = -1.96(\%\Delta P)$$

Rearrange:

$$(2.15 + 1.96)(\%\Delta P) = 4.3$$

Solve for price:

$$\%\Delta P = \frac{4.3}{2.15 + 1.96} = 1.05\%.$$

Result: Price rises by about **1.05%**.

Step 5. Solve for Quantity

Plug back into supply or demand:

Using supply:

$$\%\Delta Q_S = 2.15(1.05) - 4.3$$
$$\%\Delta Q_S \approx -2.05\%$$

Check with demand:

$$\%\Delta Q_D = -1.96(1.05) \approx -2.05\%$$

Result: Quantity falls by about **2.05%**.

Pork Market: Supply Shift from Manure Regulations

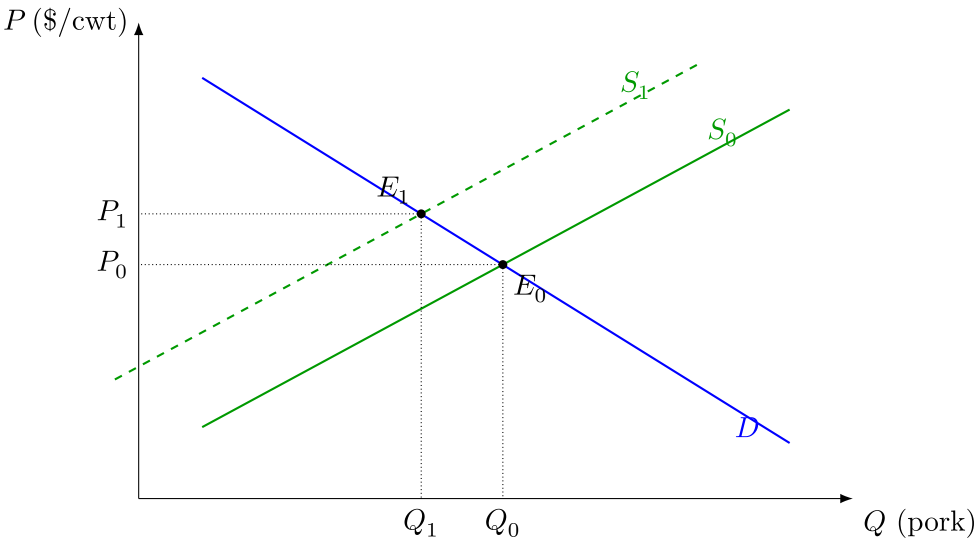

Figure 1: Manure regulations raise costs ⇒ supply shifts left ($S_0 \to S_1$): P rises, Q falls.

Interpretation

- Although regulations raised production costs by 2%, the market price rose by only about 1%. In other words, a 2% cost increase (relative to price) cannot be fully passed on to consumers.
- The remainder of the adjustment occurred through reduced output: pork production fell by about 2%.
- This reflects how elasticities distribute the burden of shocks between **price changes** and **quantity changes**.